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THEORETICAL OPTIMIZATION OF CARTRIDGE ACTUATED DEVICES

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THEORETICAL OPTIMIZATION OF CARTRIDGE ACTUATED DEVICES

The optimum cartridge actuated device for a given application may be considered the one with the "best" mechanical design consonant with the "best" ballistic performance. The "best" mechanical design often connotes the smallest or lightest device which will do the job, though other aspects of design, such as simplicity and reliability, are also involved. The "best" ballistic performance embodies the concepts of reliability and reproducibility of operation. With the propellants presently available for use in cartridges, reliability and reproducibility are intimately related to pressure level at which the device operates. Both tend to increase as the pressure level is raised. Optimization in cartridge actuated devices thus often resolves into the problem of obtaining the smallest and lightest practical device operating at the highest practical pressure level.

The gains which can be expected to accrue in size and weight reductions from optimization can be predicted from an analytical treatment of the interrelations obtaining between size and weight of the device on the one hand and the level of operating pressure on the other. A few cases will be considered here from the standpoint of what can be done toward optimization of cartridge actuated devices through adjustments of the pressure levels at which the devices operate.

THE FORCE PROBLEM

Let us consider first the case of a cartridge actuated device for which the simple and sole requirement is that the device exert a given force through a piston. Device length and piston weight will also be considered fixed. Such a device might take the form shown in Figure 1, having a circular cross-section.

The force, F, is to remain constant. It is related to the internal pressure, P, the piston area, A, and inside radius of the device, r_1, by

$$F = AP = \pi r_1^2 P \qquad (1)$$

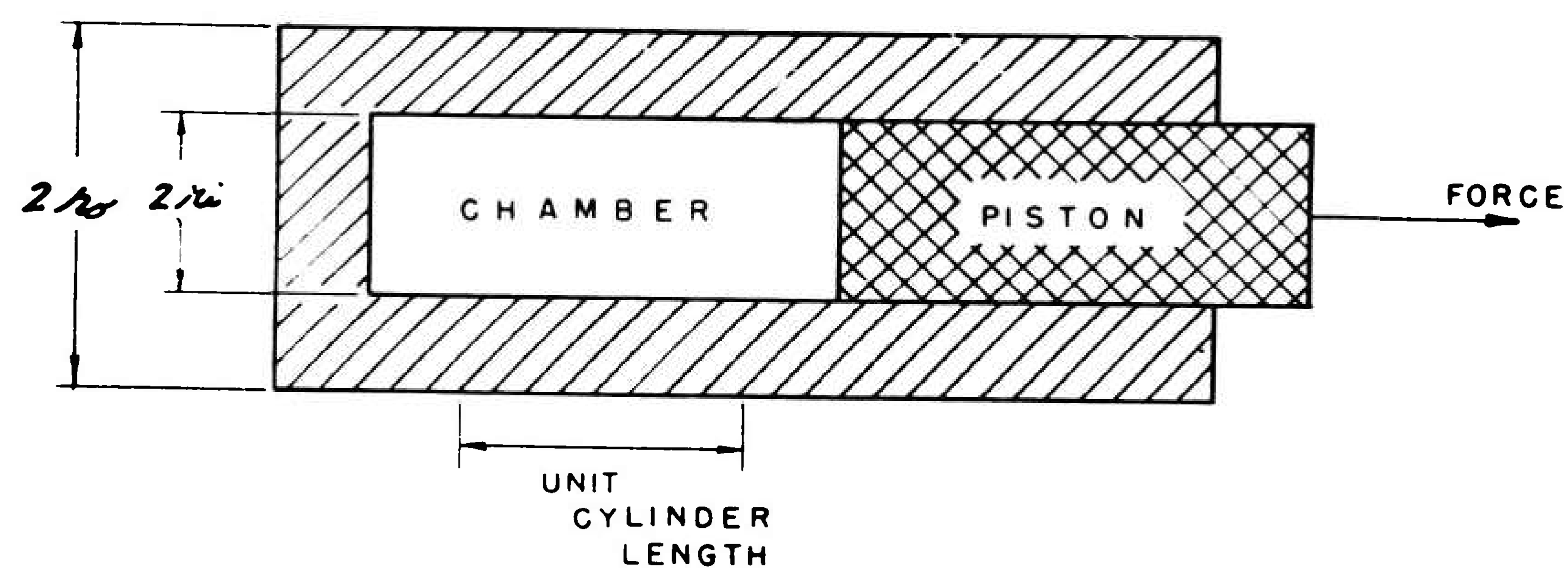


FIGURE 1

In a thick-walled cylinder with internal pressure only, the wall stresses are maximum at the inner surface and the tangential stress is usually the critical one. The tangential stress, S, is related to the internal pressure, P, and inside and outside radii, r_i by r_o by

$$\frac{P}{S} = \frac{r_o^2 - r_i^2}{r_o^2 + r_i^2} \tag{2}$$

The volume per unit length of cylinder wall, (the annulus volume per unit length), v, is given by

$$v = \pi (r_o^2 - r_i^2) \tag{3}$$

We now proceed to obtain expressions for the inner and other radii and annulus volume in terms of the internal pressure. The tangential stress, S, will be maintained constant. The absolute magnitude of this parameter in a specific device will depend on the material used for the cylinder and the margin of safety desired in the device.

From equation (1) we have

$$A = F/P \tag{4}$$

$$r_i = \left(\frac{F}{\pi P}\right)^{1/2} \tag{5}$$

Solving equation (2) for r_o gives

$$r_o = \left(\frac{1 + P/S}{1 - P/S}\right)^{1/2} r_i$$

and substituting for r_i from equation (5),

$$r_o = \left(\frac{1 + P/S}{1 - P/S} \cdot \frac{F}{\pi P}\right)^{1/2} \tag{6}$$

Substituting for r_i and r_o from equations (5) and (6) in equation (3) gives

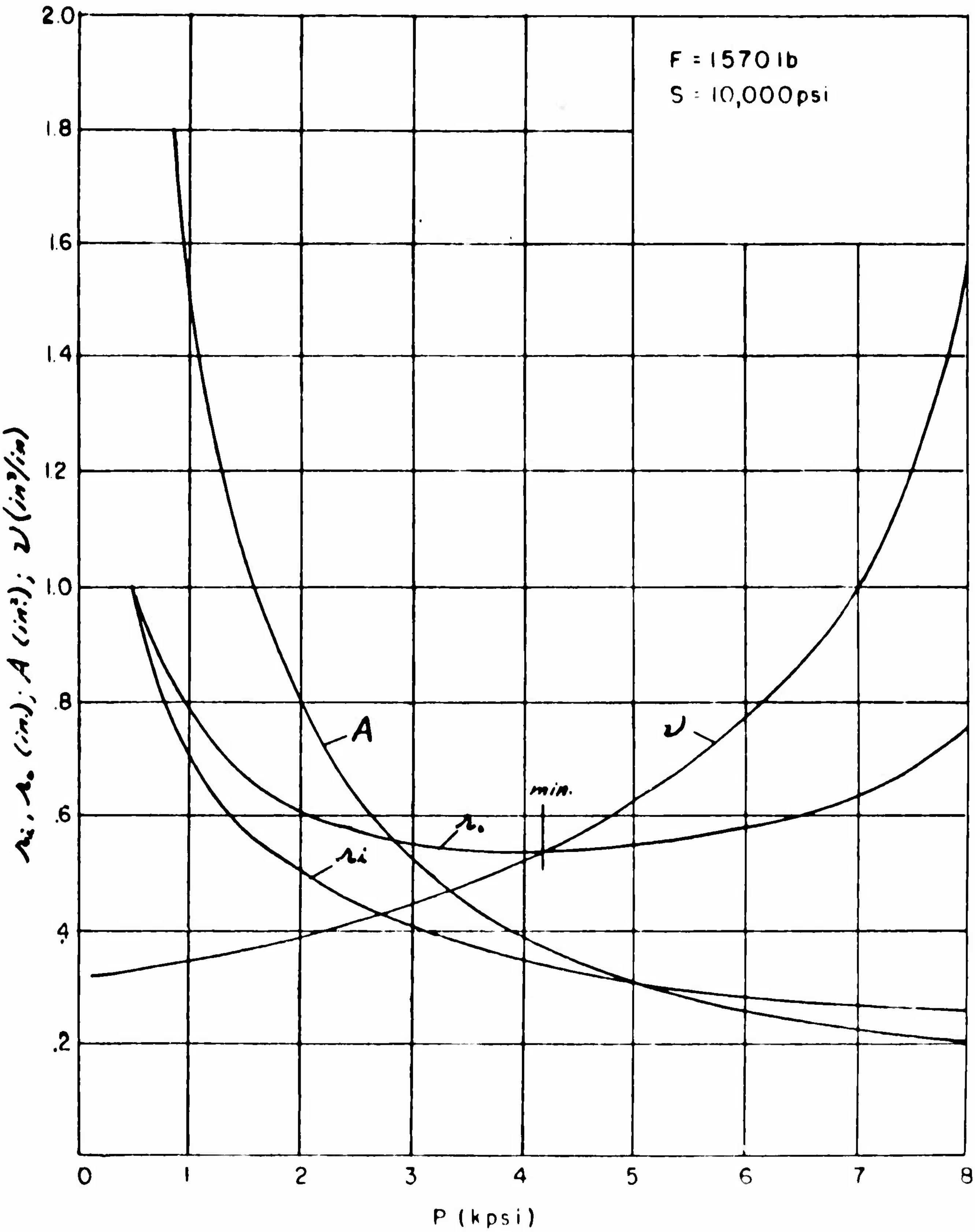
$$
\begin{aligned}
v &= \pi\left[\left(\frac{1 + P/S}{1 - P/S} \cdot \frac{F}{\pi P}\right) - \frac{F}{\pi P}\right] \\
&= \left(\frac{1 + P/S}{1 - P/S} - 1\right)\frac{F}{P}
\end{aligned}
\qquad (7)
$$

Equations (4) through (7) are graphed in Figure 2 for values of $F = 1570$ lb, $S = 10{,}000$ lb/in^2 and P varying from 500 to 8000 psi. These curves show that the optimum (smallest) overall diameter for the device, about 1 inch, occurs at a pressure of about 4000 psi, with the corresponding piston diameter about .7 inch. It can be shown that equation (6) has a minimum at $P = S/1 + \sqrt{2}$, which is at $P = 4150$ psi in this example. It will be noted also in Figure 2 that relatively large changes in operating pressure can be made either side of the optimum pressure of 4150 psi with little effect on the outside diameter. Reducing the pressure by 45% (to 2300 psi) or increasing it by 53% (to 6300 psi) increases the outside diameter by only 10%.

If, on the other hand, minimum weight is desired regardless of size, and the piston weight is kept constant, the optimum device will be as large as possible without forcing an increase in the mass of the piston. There is, in the weight sense then, no practicable design which is theoretically optimum.

THE ACCELERATION PROBLEM

A similar problem in optimization is encountered in a cartridge actuated device required to impart a given acceleration to a specified weight which is large relative to the piston weight. We will again hold the device length, piston stroke and piston weight constant. If the piston weight is negligible compared to the total accelerated weight, the difference in ballistic performance introduced by variations in piston weight as a result of changes in piston diameter will also be negligible. Use of a hollow or tubular piston will further reduce the effect of piston weight on ballistics. A device of this nature, accelerating a weight vertically, is shown in Figure 3.



F = 1570 lb
S = 10,000 psi
A
v
min.
r_o
r_i
r_i, r_o (in); A (in²); v (in³/in)
0
1
2
3
4
5
6
7
8
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
P (kpsi)

Figure 2

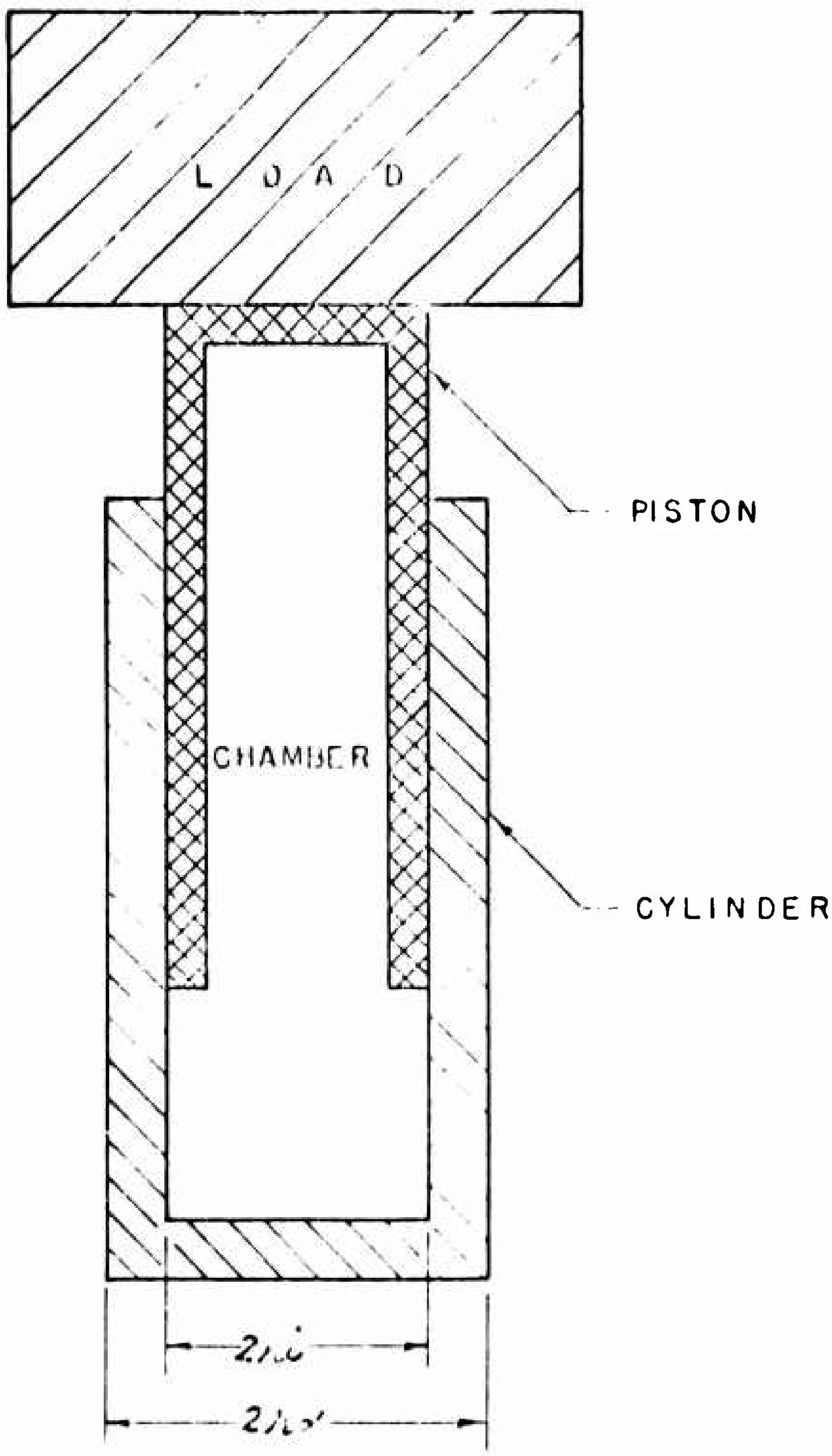


FIGURE 3

The acceleration, a, is related to the pressure, piston area, piston radius and accelerated weight, W, by

$$a = g\left(\frac{AP}{W} - 1\right) = g\left(\frac{\pi r_i^2 P}{W} - 1\right) \tag{8}$$

Since a and W are fixed, the force and AP are also constant and we have essentially the problem considered first with the requirements given in terms of different parameters. We will now develop the equations for the system in terms of W and P instead of F and P as was done in the first case.

The relation between P and S of equation (2) still applies

$$\frac{P}{S} = \frac{r_o^2 - r_i^2}{r_o^2 + r_i^2} \tag{2}$$

as does that for the annulus volume per unit length of equation (3).

$$v = \pi\ (r_o^2 - r_i^2) \tag{3}$$

The tangential stress will again be maintained constant.

From equation (8) we have

$$A = \frac{W(a + g)}{gP} \tag{9}$$

and

$$r_i = \left[\frac{W(a + g)}{\pi gP}\right]^{1/2} \tag{10}$$

From equation (2)

$$r_o = \left(\frac{1 + P/S}{1 - P/S}\right)^{1/2} r_i$$

and substituting for r_i from equation (10)

$$r_o = \left[\frac{1 + P/S}{1 - P/S} \cdot \frac{W(a + g)}{\pi g P}\right]^{1/2} \tag{11}$$

Substituting for r_i and r_o from equations (10) and (11) in equation (3) gives

$$
\begin{aligned}
v &= \pi\left[\left(\frac{1 + P/S}{1 - P/S} \cdot \frac{W(a + g)}{\pi g P}\right) - \frac{W(a + g)}{\pi g P}\right] \\
&= \left(\frac{1 + P/S}{1 - P/S} - 1\right) \frac{W(a + g)}{gP}
\end{aligned}
\qquad (12)
$$

One of the Navy Ejection Seat Catapults now in service is essentially this type of device. What does theory show that could be accomplished in the way of optimization of this catapult? The catapult gives a theoretical acceleration of 20 g's (644 ft/sec^2) to a 360 lb ejected load at a pressure of 1490 psi.

$$\left(a_g = \frac{5.07 \times 1490}{360} - 1 = 20 \text{ g's}\right)$$

At the pressure 1490 psi, the tangential stress at the inside cylinder wall is 22,800 psi computed from equation (2).

The minimum value of r_o occurs at

$$P = S/1+\sqrt{2} = 22{,}800/2.41 = 9{,}500 \text{ psi}$$

Values of the pertinent parameters for P = 1490 and P = 9,500 computed by the above set of equations are tabulated below.

P (psi)	1490	9,500
r_i (in)	1.27	.503
r_o (in)	1.355	.785
A (in^2)	5.07	.795
v (in^3)	.698	1.133
Weight of Catapult (lb)	7.5	12 (estimated)

Thus we see that the diameter of the catapult tube could be reduced approximately 40% at the expense of a 60% increase in the weight and a 6x increase in pressure. However, this type of analysis does not give us any direct information on the optimum pressure for the device from the standpoint of ballistic efficiency, reproducibility and reliability. In most applications of this type the optimum device will inevitably involve a compromise between size, weight and pressure level. In general the higher operating pressure would give more reliable ballistics. We would not recommend changing the present Navy ejection seat catapult design which has been proved in many years of use and design improvement. However, the preceding analysis will be useful in future design of similar devices.

THE KINETIC ENERGY PROBLEM

A somewhat more interesting case in optimization is that of a device required to impart a given kinetic energy to a piston. We will consider the case wherein the device length, piston length and piston stroke are functions of the piston radius and wherein the pressure and force are constant over the total piston stroke. We then have a device such as is shown in Figure 4. We will assume the device to be operating in the horizontal plane.

The kinetic energy, E, is related to the piston travel, x, piston area, A, piston radius, r_i, and pressure, P, by

$$E = xAP = \pi x r_i^2 P \qquad (13)$$

We will develop the set of equations for the system in terms of E and P, where E will be taken as a fixed value.

From equation (13)

$$r_i^2 = \frac{E}{\pi x P}$$

Let $x = \lambda r_i$, then

$$r_i^2 = \frac{E}{\pi \lambda r_i P}$$

or

$$r_i^3 = \frac{E}{\pi \lambda P}$$

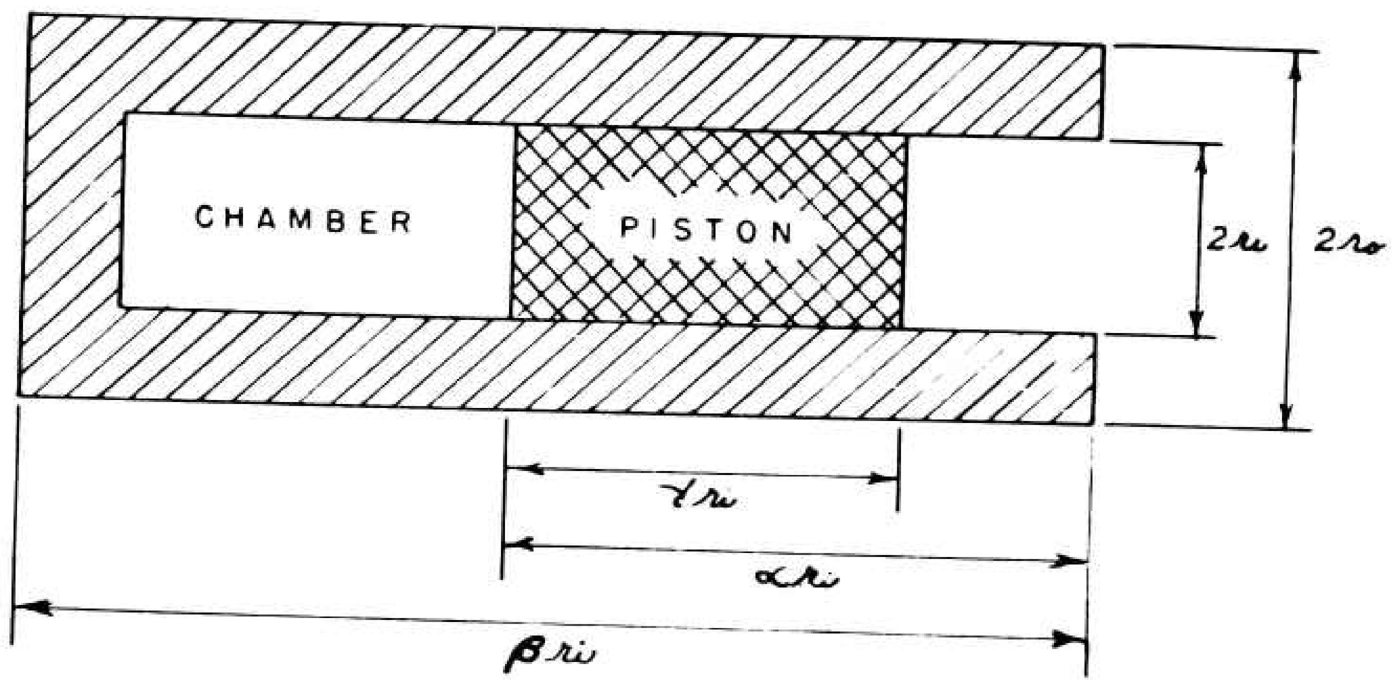


FIGURE 4

which yields $$r_i = \left(\frac{E}{\pi \chi P}\right)^{1/3} \tag{14}$$

Also, from equation (13)

$$A = \frac{E}{\chi P}$$

$$= \frac{E}{\alpha r_i P}$$

and substituting for r_i from equation (14)

$$A = \left(\frac{\pi E^2}{\alpha^2 P^2}\right)^{1/3} \tag{15}$$

The relation between P and S (again a constant) of equation (2) applies, or

$$\frac{P}{S} = \frac{r_o^2 - r_i^2}{r_o^2 + r_i^2} \tag{2}$$

From which

$$r_o = \left(\frac{1 + P/S}{1 - P/S}\right)^{1/2} r_i$$

$$= \left(\frac{1 + P/S}{1 - P/S}\right)^{1/2} \left(\frac{E}{\pi \chi P}\right)^{1/3} \tag{16}$$

The annular volume per unit length of the cylinder is, by equation (3)

$$v = \pi (r_o^2 - r_i^2) \tag{3}$$

By multiplying equation (3) by the density, ρ, and the cylinder length, L, we obtain the cylinder weight, W_c).

$$W_c = \rho L v = \pi \rho L (r_o^2 - r_i^2)$$

Let $L = \beta r_i$, and

$$W_c = \rho\pi\beta r_i(r_o^2 - r_i^2)$$

and by substituting for r_i and r_o from equation (14) and (16)

$$W_c = \frac{\rho\beta E}{\alpha P}\left(\frac{1 + P/S}{1 - P/S} - 1\right) \tag{17}$$

Denoting the piston length by h, the piston weight, W_p, is

$$W_p = \rho\pi h r_i^2$$

Let $h = \gamma r_i$ then

$$W_p = \rho\pi\gamma r_i^3$$

and by substituting for r_i from equation (14)

$$W_p = \frac{\rho\gamma E}{\alpha P} \tag{18}$$

The total device weight, W_d, (neglecting the breech weight) is then equal to $W_c + W_p$. Adding equations (17) and (18), we have

$$W_d = W_c + W_p = \frac{\rho E}{\alpha P}\left[\beta\left(\frac{1 + P/S}{1 - P/S} - 1\right) + \gamma\right] \tag{19}$$

The piston acceleration and end velocity and time to end of stroke can also be expressed in terms of E and P. We obtain first an expression for time, t, from

$$E = \frac{g}{2W_p}(APt)^2$$

from which

$$t^2 = \frac{2WpE}{gA^2P^2}$$

Substituting for A and W_p from equations (15) and (18)

$$t^2 = \frac{2E}{gP^2}\left(\frac{\rho\gamma E}{\alpha P}\right)\left(\frac{\alpha^2 P^2}{\pi E^2}\right)^{2/3}$$

and

$$t = \left(\frac{2\rho\gamma}{gP}\right)^{1/2}\left(\frac{\alpha^{1/2}E}{\pi P}\right)^{1/3} \tag{20}$$

The acceleration, a, is

$$a = \frac{gAP}{W_p}$$

Substituting for A and W_p from equations (15) and (18)

$$a = gP\left(\frac{\alpha P}{\rho\gamma E}\right)\left(\frac{\pi E^2}{\alpha^2 P^2}\right)^{1/3}$$

$$= \frac{gP}{\rho\gamma}\left(\frac{\alpha\pi P}{E}\right)^{1/3} \tag{21}$$

The velocity, v, is

$$v = \frac{gAPt}{W_p} = at$$

Substituting for a and t from equations (20) and (21)

$$v = \frac{gP}{\rho\gamma}\left(\frac{\alpha\pi P}{E}\right)^{1/3}\left(\frac{2\rho\gamma}{gP}\right)^{1/2}\left(\frac{\alpha^{1/2}E}{\pi P}\right)^{1/3}$$

$$= \left(\frac{2g\alpha P}{\rho\gamma}\right)^{1/2} \tag{22}$$

Let us now consider the optimization of a hypothetical device of the type shown in Figure 4 for which the following requirements are given:

$E = 5{,}000$ in/lbs

$S = 20{,}000$ psi

$\alpha = 6$ or $x = 6r_i$

$\beta = 10$ or $L = 10r_i$

$\gamma = 4$ or $h = 4r_i$

$\rho = .1$ lb/in^3

We can compute values of the several parameters for varying values of pressure independently from the set of equations developed above. However, if we wish to compute all, or most, of the parameters, the computations will be simplified considerably by rewriting the set of equations in terms of r_i, computing r_i from equation (14) then proceeding by the following equivalent equations:

$$A = \pi r_i^2$$

$$r_o = \left(\frac{1 + P/S}{1 - P/S}\right)^{1/2} r_i$$

$$W_c = \rho\pi \quad (r_i r_o^2 - r_i^3)$$

$$W_p = \rho\pi\gamma r_i^3$$

$$W_d = W_c + W_p$$

$$t = \left(\frac{2\rho\gamma\alpha}{gP}\right)^{1/2} r_i$$

$$a = \frac{gP}{\rho\gamma r_i}$$

$$v = at$$

The variations of the several parameters as functions of pressure over the range 1,000 to 10,000 psi for our hypothetical device are shown in Figures 5 and 6. It will be seen in Figure 5 that both the outside radius and the total device weight (less breech) reach minima at about 6,000 psi. It can be shown that equation (16) has a minimum at $P = \frac{S}{2}(\sqrt{13} - 3)$, which in this case is at $P = 6060$ psi. Equation (19) has a minimum at $P = \frac{S}{1 + \sqrt{\frac{2\beta}{\gamma}}}$ or, in our example at $P = 6170$ psi. Since r_o is a minimum at $P = \frac{S}{2}(\sqrt{13} - 3)$, the pressure at minimum r_o depends only on the selected value of S. Minimum r_o and W_d occur at the same pressure when $\frac{S}{2}(\sqrt{13} - 3) = \frac{S}{1 + \sqrt{2\beta/\gamma}}$ or when $\beta/\gamma = 1/2\left(\frac{2}{\sqrt{13} - 3} - 1\right)^2 = 2.65$. In the example considered here $\beta/\gamma = 2.5$.

Piston length, stroke and weight and cylinder length, by assumption, are functions of inside radius, and they all decrease, as does the inner radius, with increasing pressure. In Figure 6, we see that while the piston stroke and time of stroke decrease with increasing pressure, the acceleration and velocity increase with pressure.

There is then an optimum pressure, outside diameter and total weight for the device. All other parameters, except cylinder weight, acceleration and velocity, continue to decrease in magnitude (but at decreasing rates) as the pressure is increased above the optimum value from the standpoint of diameter and total weight of the device. However, since the outside diameter and total weight are relatively insensitive to pressure changes in the region of their minima, we can design a device to operate over a rather broad range of pressures and yet have the device near the optimum in diameter and total weight. This could conceivably be an advantage where compromise must be made with other parameters, either mechanical or functional. For example, the acceleration can be approximately halved by operating at 4,000 psi pressure instead of the optimum 6,000 psi while outside diameter and total weight are increased by only about 3 and 7% respectively.

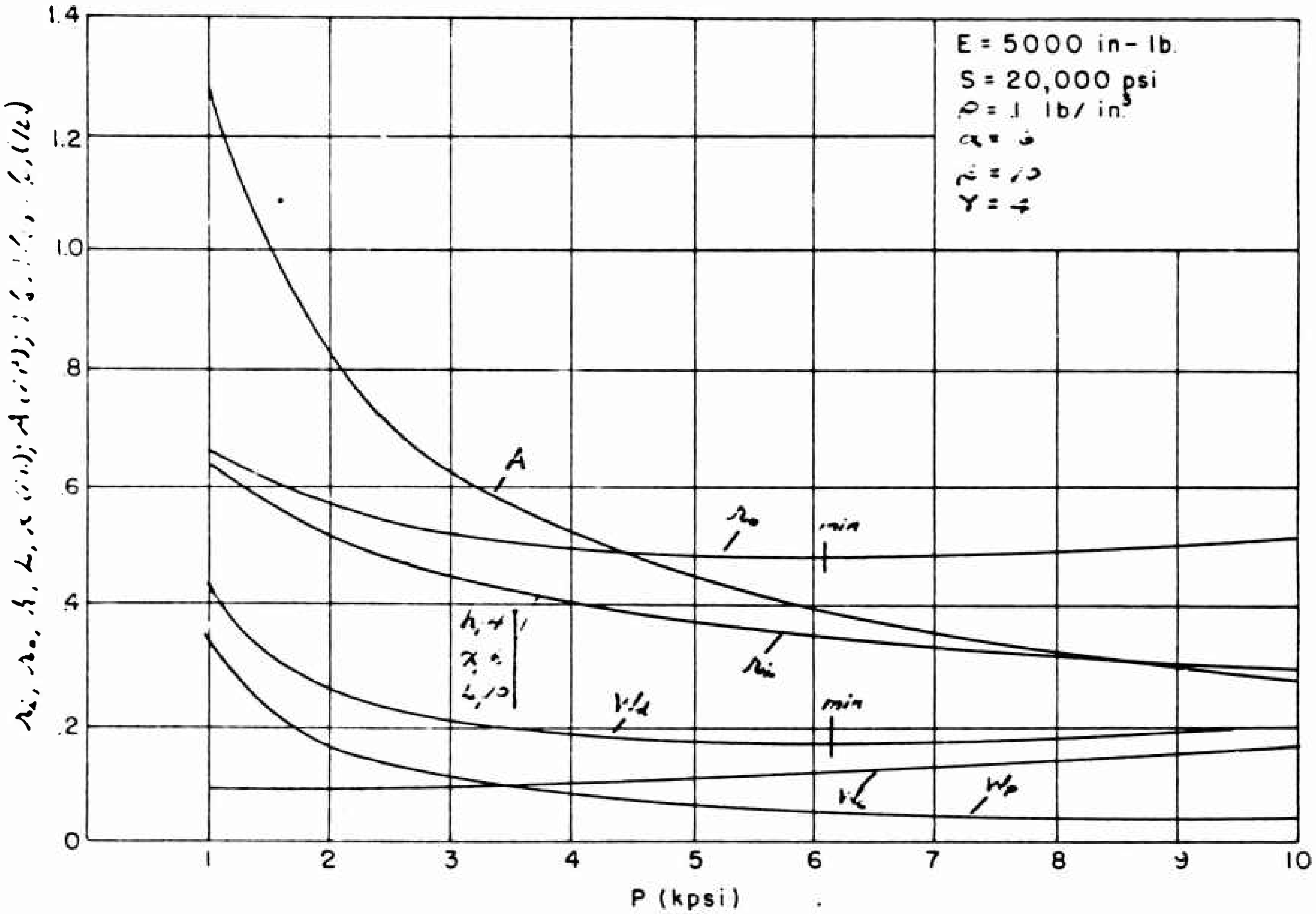
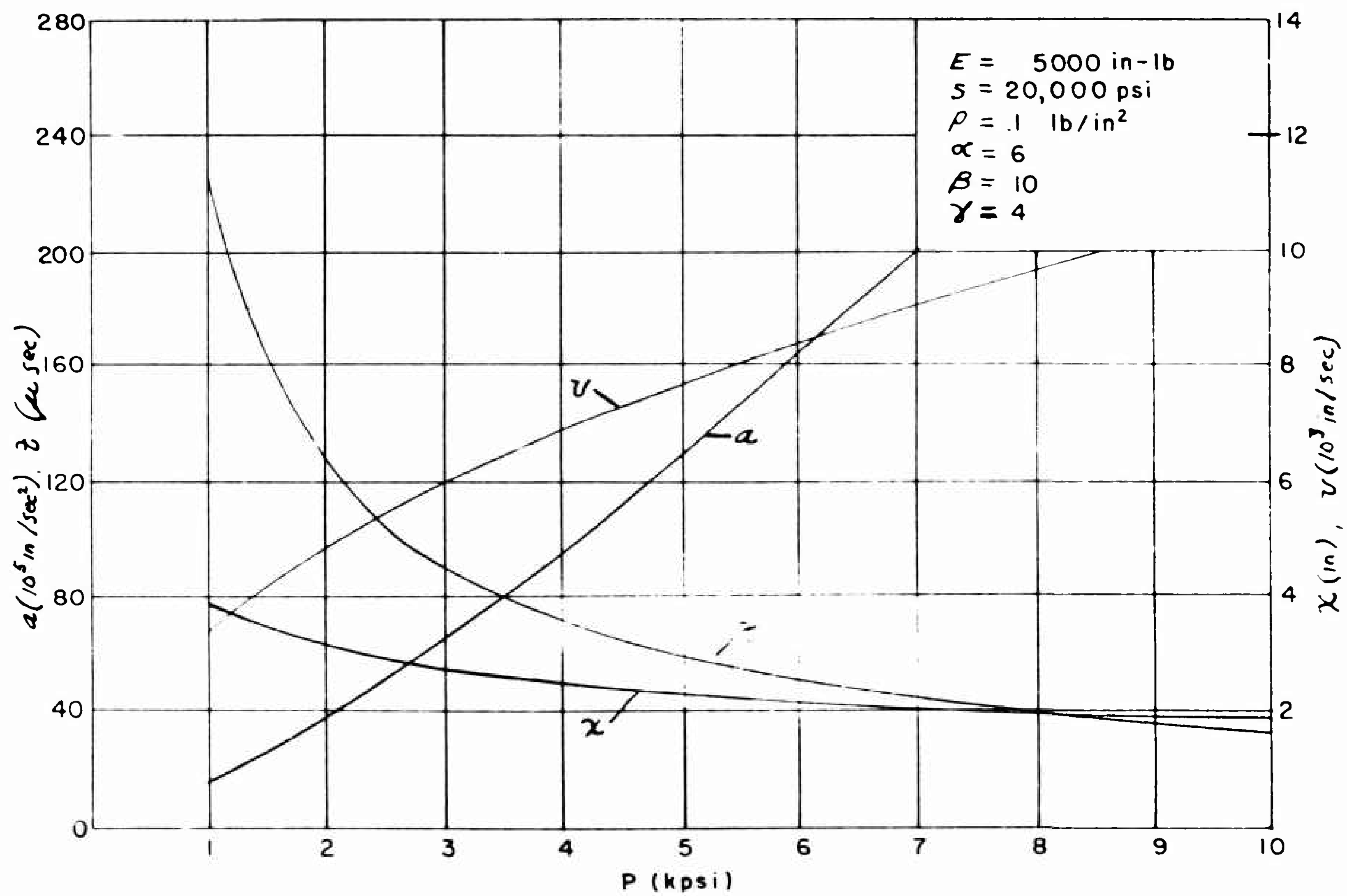

E = 5000 in-lb.
S = 20,000 psi
P (kpsi)
1.4
1.2
1.0
.8
.6
.4
.2
0
1
2
3
4
5
6
7
8
9
10
A
min
min

Figure 5



E = 5000 in-lb
S = 20,000 psi
ρ = .1 lb/in²
α = 6
β = 10
γ = 4
a(10⁵ in/sec²), t (μ sec)
χ (in), v(10³ in/sec)
P (kpsi)
v
a
t
χ

Figure 6

SUMMARY

The examples considered here are sufficient to illustrate the analytical approach to the optimization of cartridge actuated devices and to show that the method is capable of predicting the gains in size and weight reductions which can be expected from optimization in specific devices. In general terms we see that; (1) definite minima in size and/or weight may exist in some cases, while in others there are no practical minima; (2) minimum weight and size may or may not occur at the same operating pressure; and (3) there may be considerable latitude in selecting the operating pressure while yet having the device near optimum in size and/or weight. The analytical approach promises to be an effective tool in the optimization of cartridge actuated devices.

NOMENCLATURE

a	acceleration
A	piston area
E	kinetic energy
F	force
g	acceleration of gravity, gravity constant
h	piston length
L	cylinder length
P	pressure
r_i	inside radius of cylinder, piston radius
r_o	outside radius of cylinder
S	tangential stress
t	time
v	velocity
W	accelerated weight
W_c	weight of cylinder
W_p	weight of piston
W_d	weight of cylinder plus piston
x	piston travel
α	ratio of x to r_i
β	ratio of L to r_i
γ	ratio of h to r_i
ν	volume per unit length of cylinder
ρ	density

U. S. Naval Weapons Laboratory
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From: Commander, U. S. Naval Weapons Laboratory
Dahlgren, Virginia
To: Distribution List of Power Cartridge Handbook

Subj: Theoretical Optimization of Cartridge Actuated Devices

Encl: (1) Technical Memorandum No. W-22/59

1. Enclosure (1) is a preliminary report on a study of the analytical approach to the optimization of cartridge actuated devices. It is an attempt to demonstrate, by a few representative examples, the method and effectiveness of this approach to the problem of optimization.

2. The memorandum is punched for inclusion in the Power Cartridge Handbook if so desired.

A. R. FAUST

~~D. W. STONER~~ Stoner
By direction